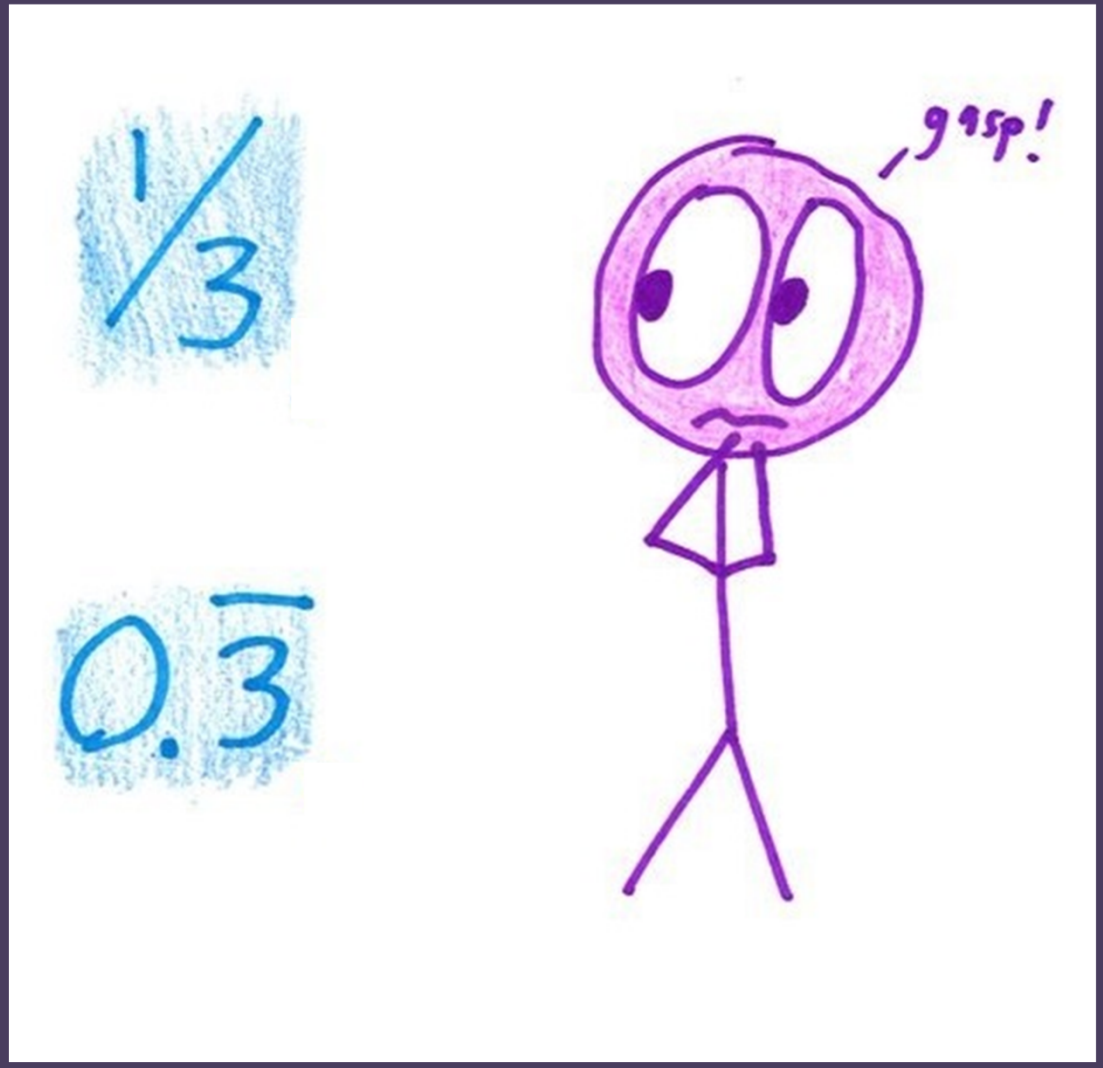
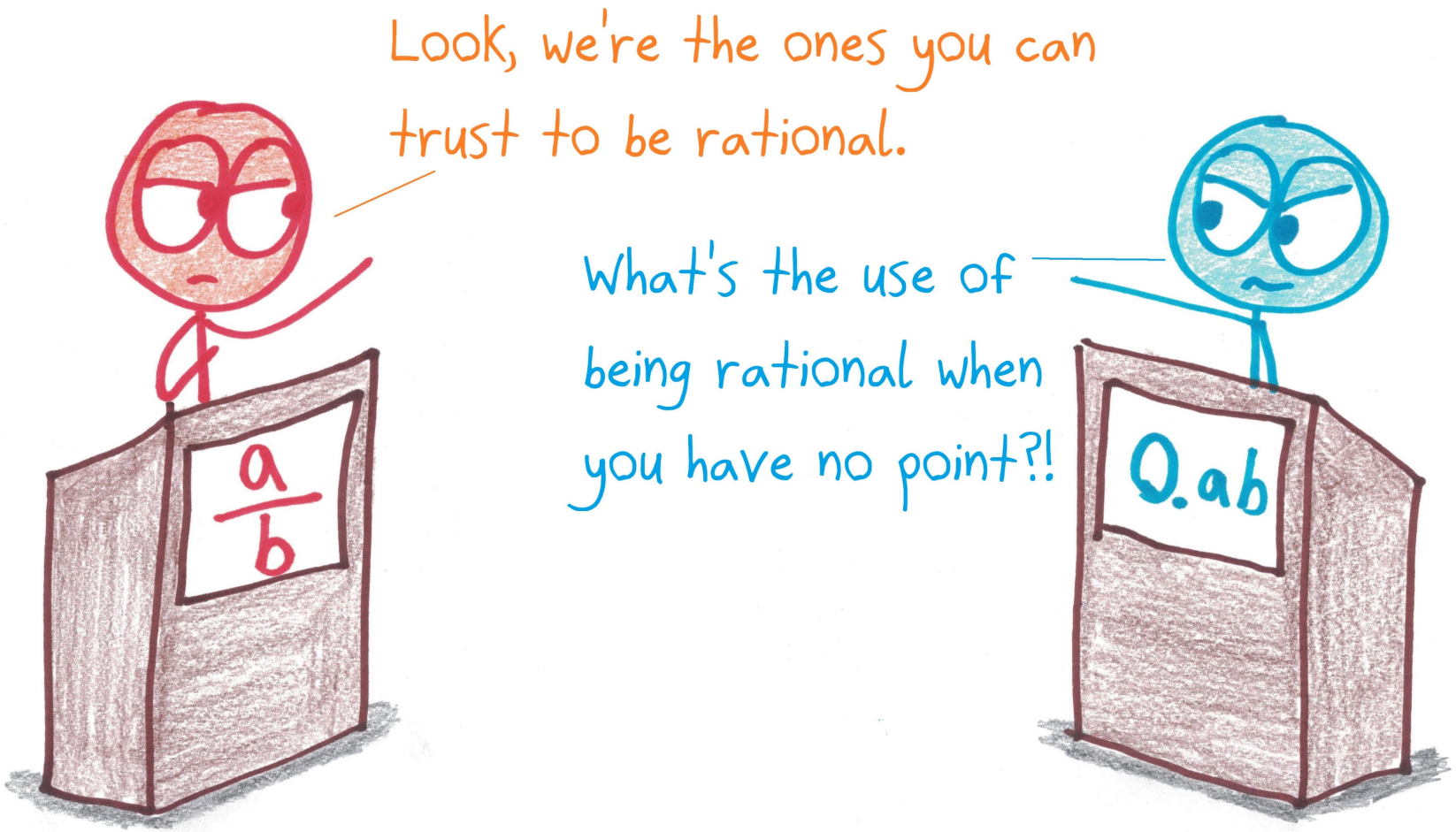
Fractions vs. Decimals

Ben Orlin

Math with Bad Drawings



TEAM FRACTION VS. TEAM DECIMAL



Name: _____

Remember to find the common factors first.

Simplify the fractions which means reducing fractions to their lowest terms. First you will need to know what the largest common factors are for the demonimator and numerator.

1. $\frac{12}{18} =$

2. $\frac{30}{36} =$

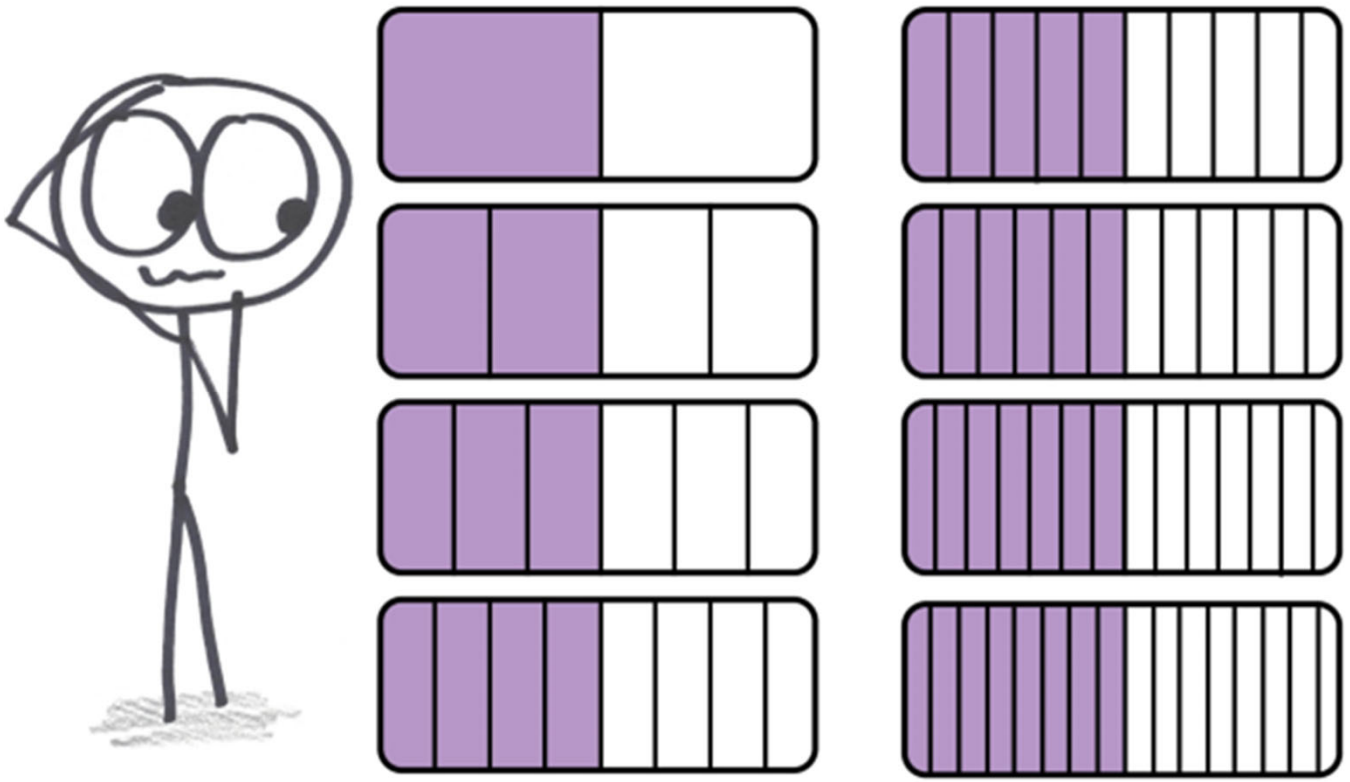
3. $\frac{3}{12} =$

4. $\frac{15}{25} =$

5. $\frac{3}{18} =$

6. $\frac{12}{24} =$

Give five fractions that are bigger than 1/2, but smaller than 1.



Challenge: Can you put them in ascending order?

Which is bigger: 1/5 or 1/6?

Give five fractions that fall between the two of them.

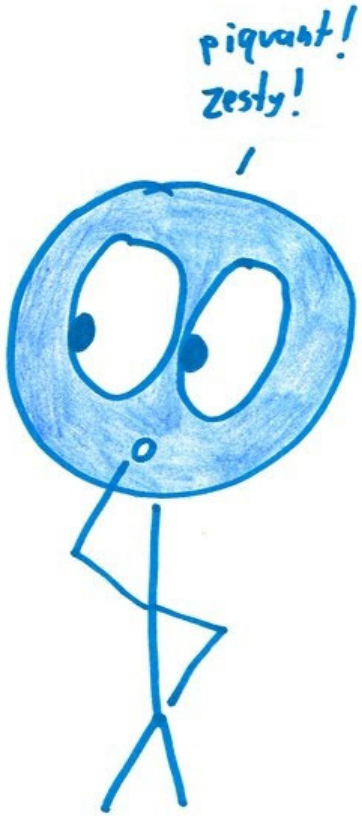
Which is bigger?

$\frac{17}{30}$ $\frac{18}{31}$

Which is bigger?

$$\frac{3997}{4001} \text{ or } \frac{4996}{5001}$$

Hint: They differ by less than 0.00000005.



Well, let's consider the missing pieces:

$\frac{3997}{4001}$ vs. $\frac{4996}{5001}$

$\frac{3997}{4001}$ falls short of 1 by $\frac{4}{4001}$

$\frac{4996}{5001}$ falls short of 1 by $\frac{5}{5001}$

So which missing piece is bigger?

This is where it gets silly.

(And by silly, I mean awesome.)

That's because each of these missing pieces is slightly short of 1/1000.

$$\frac{4.001}{4001} = \frac{1}{1000}$$

$$\frac{5.001}{5001} = \frac{1}{1000}$$

So let's put it together.

$\frac{0.001}{4001}$ is larger than $\frac{0.001}{5001}$,

so $\frac{4}{4001}$ is smaller than $\frac{5}{5001}$,

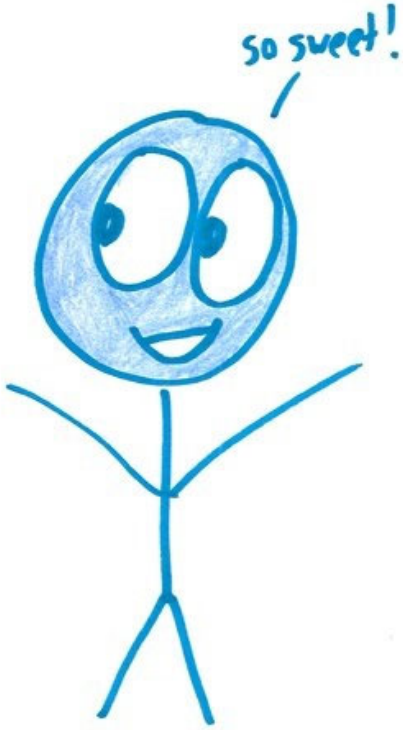
so $\frac{3997}{4001}$ is larger than $\frac{4996}{5001}$.

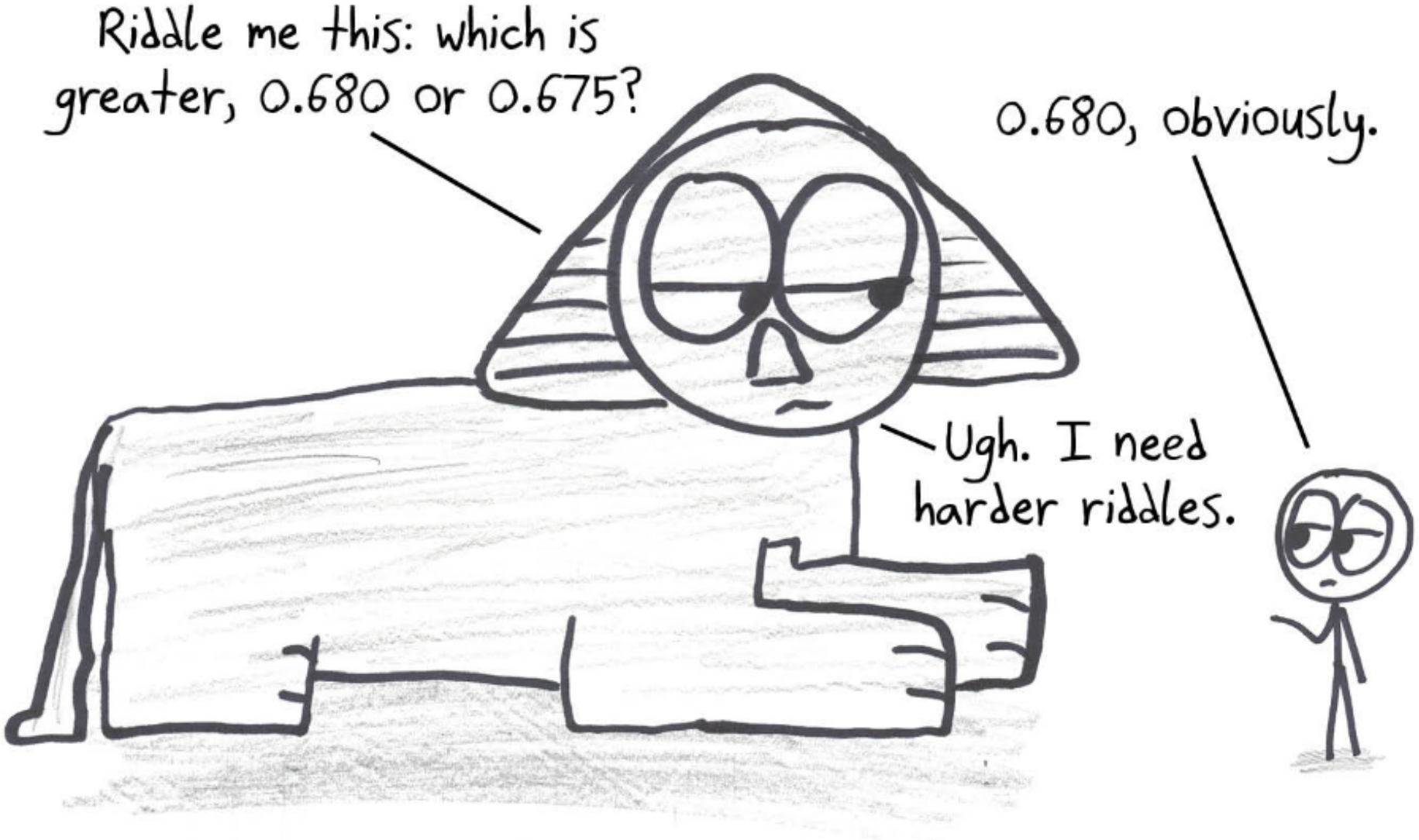
(Because the larger the missing piece, the smaller the remainder.)

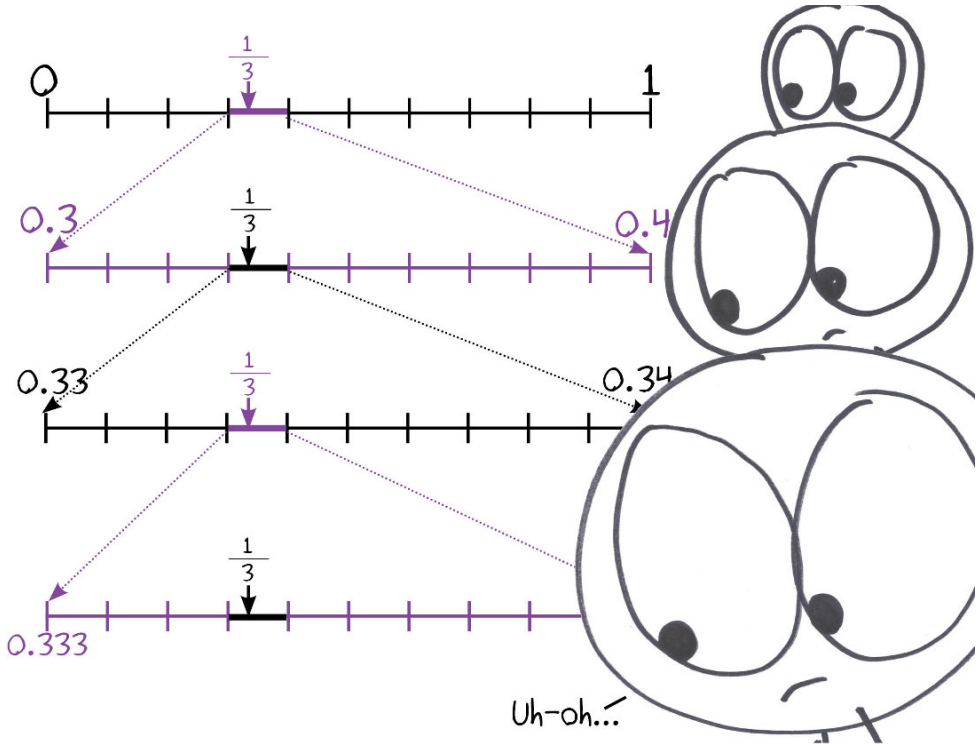
A quick calculator cameo confirms:

$$\frac{3997}{4001} \approx 0.99900025$$

$$\frac{4996}{5001} \approx 0.99900020$$




Riddle me this: which is greater, 0.680 or 0.675?
0.680, obviously.
Ugh. I need harder riddles.



Decimal-Crazed Lunatics

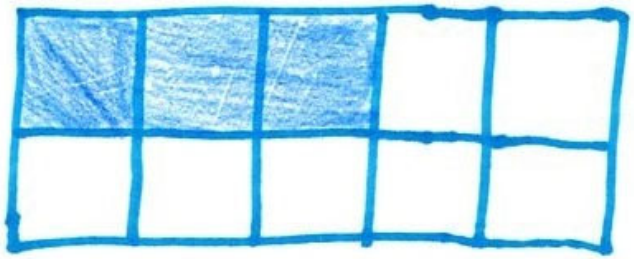
Ben Orlin Education, Math March 23, 2016 1 Minute

I had to hit the big red STOP EVERYTHING button in class the other day, because my students were acting unhinged.

Instead of $\frac{1}{3}$,

they were writing $0.\overline{3}$.

That's right. Not 

but 

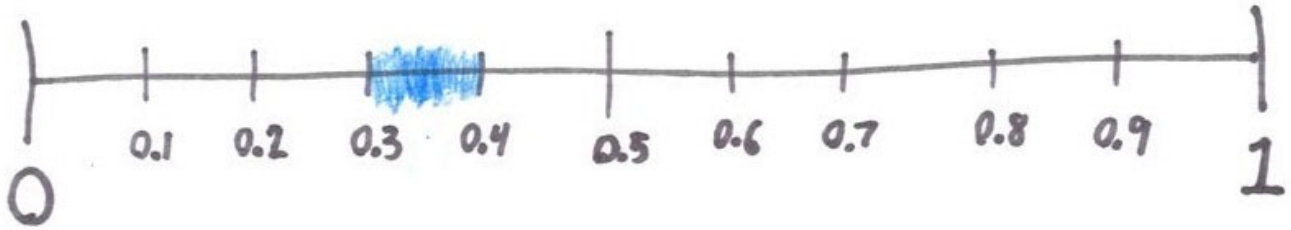
+  +  +  ...

I have just one question.

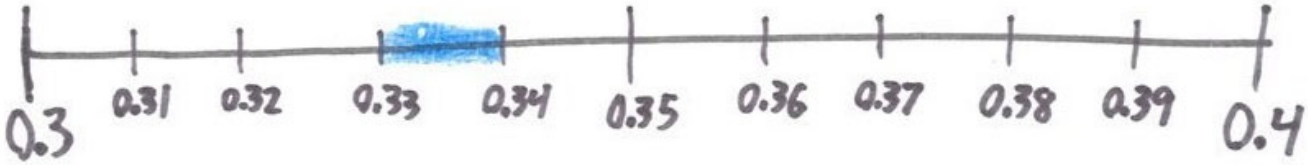
I mean, I know why.
It's because decimals
feel like numbers, and
fractions don't.

(Never mind that decimals were invented only once, less than a millennium ago, while fractions go back to the dawn of history, and were discovered independently by every mathematical culture.)

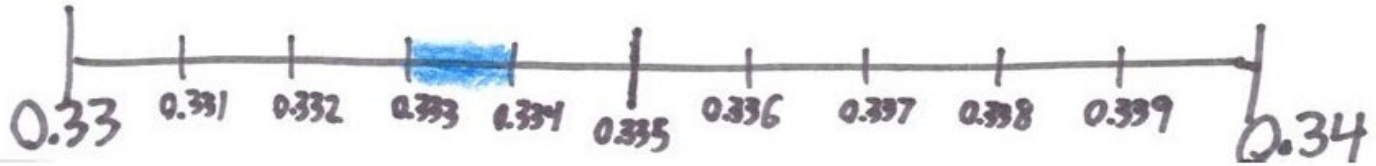
Decimals split every unit into 10 pieces.



And each of those has 10 pieces.



And each of those . . .



But no matter how deep you go,
no matter how fine you slice,
no matter how many decimal places,
you never quite get 1/3.

0.33333333333333333...

still not quite right

Fractions are more flexible.

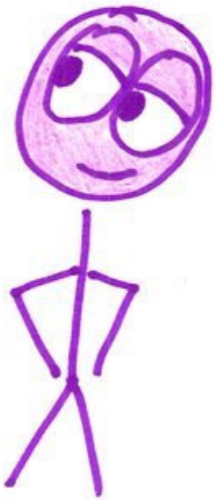
We don't have to use 10 pieces.

We can use, say, 3.

It's a choice between $\frac{1}{3}$

and $\frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \frac{3}{10{,}000} + \frac{3}{100{,}000} + \ldots$

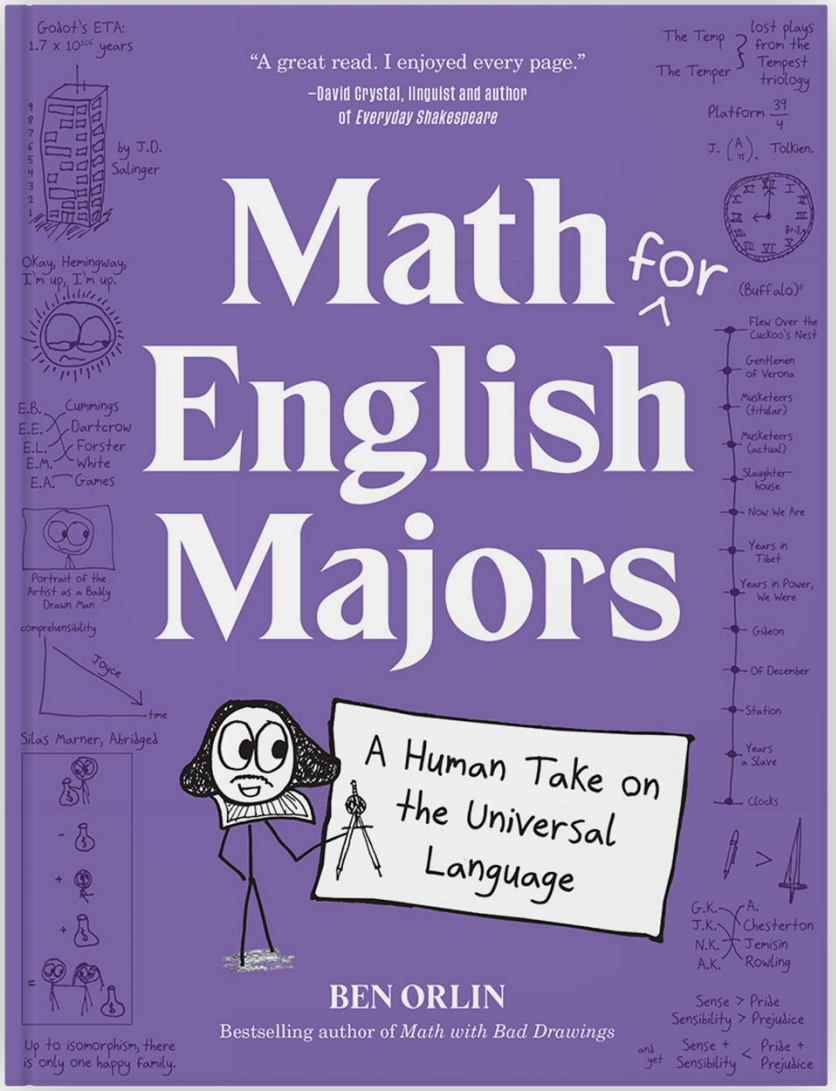
Is that choice really so hard?



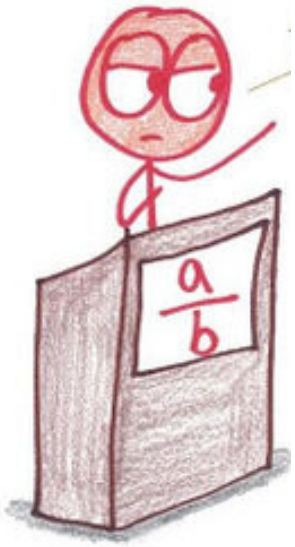
“Imagine you speak two languages,” I’d say. “One of them has a word for ‘door.’ The other does not: you just start saying ‘tuk-tuk-tuk-tuk-tuk…’ and repeat that syllable forever.”

“When you need to talk about doors, which language are you going to use?”

The classroom would erupt with a chorus of “tuk-tuk-tuk-tuk,” and I knew that Team Decimal had won this round.



Which is better for discussing…



Rough measurements (e.g., in the garden)

Size **comparisons** (e.g., greater distance)

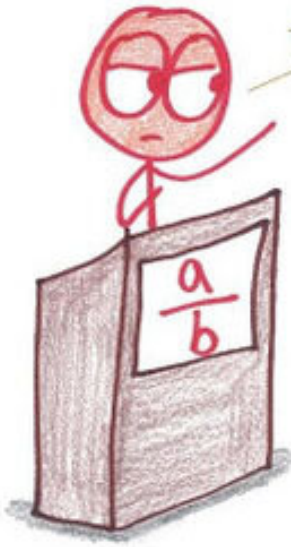
Irrational numbers (e.g., rough size of pi)

Precision in measurement (e.g., possible error of a scale)

Probabilities (e.g., in a card game)

Abstract relationships (e.g., gravity vs. mass)

Which is better for discussing…



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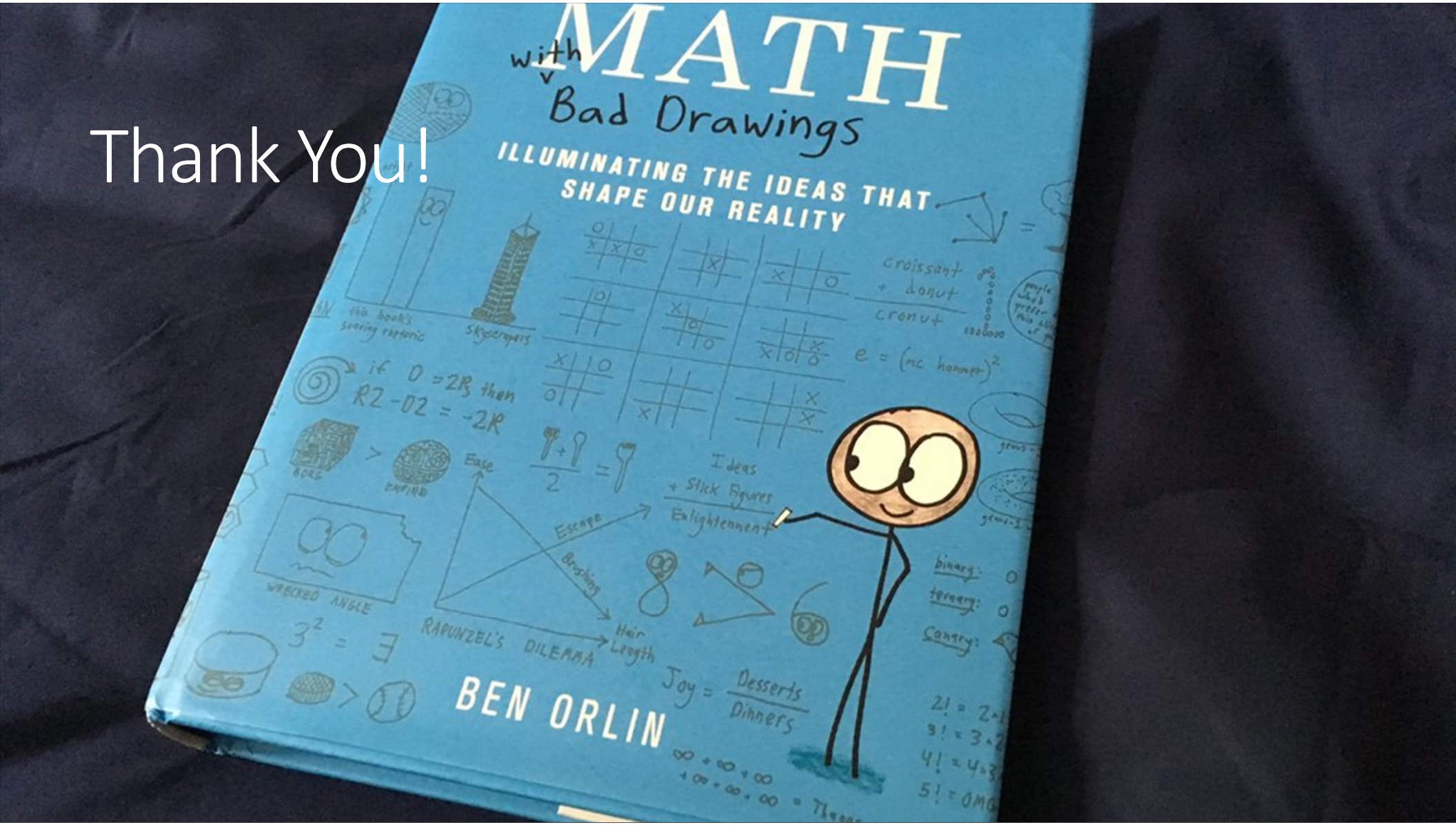
By and large, decimals are for measurement, and fractions are for theory.

Thank You!


with
MATH
Bad Drawings
ILLUMINATING THE IDEAS THAT SHAPE OUR REALITY
BEN ORLIN