CH. 4: THE SWINDLER'S COIN

Everyone makes mistakes. Only fools stand by them.

"I'm flipping a coin," the teacher said. "Tell me the probability that it comes up heads."

"Is it a fair coin?" the student asked.

"Yes," the teacher said. "I promise."

"Then the probability is 50%."

The student heard the *plink* of the flipped coin, and the slap as the teacher caught it. "Heads. Let's play again."

"50%," the student said. The coin spun through the air.

"Heads again," the teacher called. "Keep playing."

"50%," the student said. *A coin has two sides, so the probability of heads is 1 in 2. Easy,* she thought to herself.

"Heads again."

The conversation continued. “Another heads,” the teacher called. “We’re up to 30. What’s the probability that the next one is heads?”

“You’ve gotten really lucky, but the probability of heads is still 50%.”

The teacher shook her head. “You are a fool indeed.”

“But that’s how coins work,” the student said. “You told me it’s a fair coin, so the probability is 50%. All these heads have just been a wild coincidence.”

“Tell me,” the teacher said. “What is the probability of 30 heads in a row?”

“Um… 1 in a billion, more or less.”

“And there’s no chance that I’m swindling you, is there?”

“Well…” the student said. “It’s very unlikely. Not only would you have to lie to me, but you’d need to be able to manipulate coin tosses. Maybe there’s a 1 in a million chance.”

“1 in a million is quite different from impossible,” the teacher said.

“I guess.” The student shrugged.

“Now,” the teacher continued, “what is the probability that the next flip will be heads?”

The student paused. “I’m not sure. I thought it was 50%... but the chance of your tricking me is still 1000 times more likely than that you’re doing it honestly.”

“So is it 50%, or not?”

“It’s not,” the student concluded. “That’s a swindler’s coin.”

The teacher smiled. “Very good.”

The student’s face began to grow hot. “So you were lying to me this whole time! Why would you do that? You said it was a fair coin, and I trusted you.”

“As you should,” the teacher said, “up to a point. When you witness something that defies all of your assumptions about the world, you must learn to question those assumptions. To do otherwise is to disappear inside your own head, and ignore the world of evidence knocking at your door. You become one of three things: A dreamer, a fool, or a stubborn theoretician. And I’ve never had much luck telling the three apart.”

“You could have just told me that,” the student said. “You didn’t have to trick me.”

The teacher laughed. “How could I teach you about falsehoods, if I spoke only truths?”

CH. 4: ~~DISCUSSION~~ RANT

Some probability texts ask a similar question: *"If a fair coin is tossed 50 times, and comes up heads each time, what is the probability that it comes up heads on the 51st toss?"* The "correct" answer is ½. A fair coin *always* has a probability ½ of coming up heads, because that's how we define "fair."

But guess what? If a coin comes up heads 50 times in a row—a 1-in-a-quadrillion event—then that ain't no fair coin. The question could be paraphrased: "If I tell you a coin is fair, and then overwhelming evidence accumulates to the contrary, would you still believe me?" And the "correct" answer would be: "Yes, because I never reconsider my assumptions."

For probability to be useful, it ought to stay anchored in practice. We shouldn't cling to invalidated assumptions or now-obsolete frameworks. We shouldn't keep telling ourselves the emperor is clothed.

CH. 4: QUESTIONS

1. Suppose you're watching someone flip coins, and they keep getting heads. How many heads in a row would it take for you to believe that they're cheating somehow?

2. Suppose you're hearing somebody report the results of their coin flips (but you can't see for yourself). How many heads in a row would it take for you to believe they're lying?

3. Suppose you're flipping coins by yourself. How many heads in a row would it take for you to believe that someone is controlling the flips somehow?

4. Did you give the same answer to #1, #2, and #3? If not, what key factor(s) determine your willingness to believe in the coin's fairness?

5. What is Bayes' Law, and what does it have to do with this whole discussion?