CH. 3: RIDDLE OF THE ODORLESS INCENSE

See the world as it is, not as you imagine it to be.

The teacher led the student to a blind vendor who sold two types of incense, identical in appearance. "This one he calls *Forest*," the teacher said, holding up a speckled brown bundle that smelled of sandalwood and pine. "And this one he calls *Tea Garden*," the teacher said, holding up another.

The student sniffed the *Tea Garden*. "It has no odor," she said.

"Ah, a shame," the teacher said. "Only some people can smell *Tea Garden*."

The student shrugged. "Well, let's not buy any of that one."

"He sells them in bags of two," the teacher continued. "But he does not pay attention to which incense goes in which bag."

The blind vendor had a massive crate of incense, with the two types mixed together. His hands dove in and out, grabbing another stick with each motion, and throwing it into a plastic bag. When a bag contained two sticks, he set it aside for sale.

"I'll just smell the bags before we buy them," the student said. "And we'll only buy ones where I detect *Forest*."

"Ah," the teacher said. "But won't we end up paying for lots of *Tea Garden*, too?"

"The bags we'll pick already have one *Forest*," the student said. "So there's a 50% chance they'll contain another *Forest*, and a 50% chance they'll contain a *Tea Garden*. That seems worth the risk."

"Then let's buy 120 bags," the teacher said. "Start choosing."

When they got home, the teacher put her to work immediately. "Smell each stick of incense, one by one," she said. "Then make two piles of bags. The first is for those with two *Forest*. The second is for those with only one *Forest*." The teacher smiled. "And we shall see which pile is larger."

"They'll be the same size," the student said. "Like I said—there's a 1 in 2 chance the second stick is *Forest*, and a 1 in 2 chance that it's *Tea Garden*."

"But how do you know which is the second stick?" The teacher giggled and walked off.

By the time she finished sorting, the student had grown convinced something was wrong. The first pile—with two-*Forest* bags—held only 41 bags. The other pile—the one-*Forest*, one-*Tea Garden* bags—stood almost twice as high, with 79 bags.

"The vendor cheated us," the student seethed. "He deliberately gave me extra *Tea Garden*," the student said. "Only half of our bags should have *Tea Garden*. But instead, 2 out of 3 do."

They returned the next day to the vendor's stall. "Don't just watch him," the teacher said. "Help fill the bags yourself. You must see the incense as the vendor does, not as the customer does."

So the student began to fill bags, smelling each stick of incense as she grabbed it from the crate, and noting the order.

After half an hour, she exclaimed suddenly, "I get it! There are four types of bags."

"Four?"

"Yes. There's *Forest* plus *Forest*. And *Tea Garden* plus *Tea Garden*. And then, there are two more possibilities. There's *Tea Garden* plus *Forest*, and there's *Forest* plus *Tea Garden*."

"Those last two," the teacher said, "aren't they the same?"

"The bags look the same when you're done," the student said, "but they're not *created* the same. The process for each one is different.

"When we came yesterday," the student continued, "we eliminated one of the four possible bags—the ones with only *Tea Garden*. That left three other types of bags. So of the ones we brought home, the double-*Forest* should only be 1 in 3. I thought I'd only picked bags where the first stick was *Forest*, but that was wrong. In some of the bags I picked, *Forest* was the second stick."

When they arrived home, the student lit a stick of incense and sat with her eyes closed. "This *Forest* smells different than it did yesterday," she said.

The teacher smiled. "I'm not surprised," she said. "That's *Tea Garden*."

CH. 3: DISCUSSION

This is a classic—and very tricky—problem. Usually, it's presented the following way: "If a two-child family has at least one daughter, what is the probability that *both* their children are daughters?" Lots of people make the same mistake as the student in the story. The problem is an interesting introduction to the idea of "sample space," because it requires careful thinking but almost no computation.

Sample space, by the way, is nothing but a fancy term for "list of possibilities." The trick is that you've got to list possibilities that are equally likely. The student's original sample space—*F + F*, *TG + TG*, and "one of each"—was flawed, because the last item was twice as likely as either of the other two.

Technically, the incense scenario is slightly different from the daughter scenario. If you have a daughter, the probability your next child is also a daughter is precisely 50%. But if our crate of incense starts out with a 50-50 mix, then once we pick out a stick of *Forest*, there's less *Forest* than *Tea Garden* remaining. So our probability is just below 50%. (Luckily, if there are thousands of sticks of incense, as in the story, then this change is barely noticeable.)

CH. 3: QUESTIONS

1. Suppose the student picks a bag, then picks a random stick from inside and smells it alone. It's *Forest*. What's the probability that both sticks in the bag are *Forest*?

2. How is the scenario in #1 different from the scenario in the story? Does this matter?

3. *A case study in assumptions*: The student picks up a bag of two incense sticks. "Does this have any *Forest* in it?" she asks the vendor. He says yes. What is the probability that both sticks are *Forest*?

4. The vendor is now making sure that there are precisely two *Forest* sticks in every bag. But the bags now vary in size. Some have two sticks, but some have more. He just fills them, one stick at a time, until he reaches the second *Forest* stick. Then he moves on to the next bag. So by definition, the last stick he puts into each bag must be *Forest*. What's the probability that the second-to-last stick in the bag is also *Forest*?

5. Suppose you're picking sticks of incense, one after the other, until you obtain one of the following sequences: two straight *Forest*, or a *Tea Garden* followed by a *Forest*. When you hit one of those sequences, you stop picking. By definition, the last stick you pick must be *Forest*. But what's the probability that the second-to-last stick is also *Forest*?