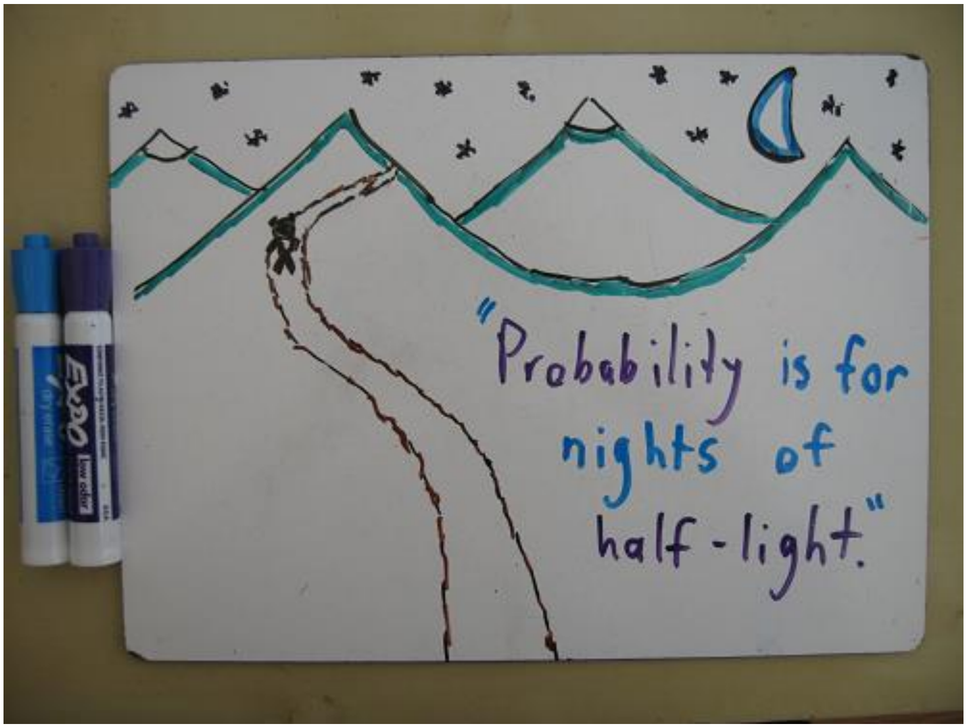

CH. 1: THE BEAR IN THE MOONLIGHT

A probability is an expression of uncertainty.

It was a half-moon that night. The student and the teacher could see a shadowy, white-chested figure lumbering down the mountain path.

"Is that a bear?" the student gasped.

The teacher nodded calmly. "It may be. Or, it may be one of the children from the village, disguised as a bear, hoping to scare his friends."

"Well, which is it?" the student hissed. "A deadly bear, or an innocent child?"

"Let us each determine the probability that the figure is a bear," the teacher said. "Then we shall share our answers with one another."

After a pause, the student whispered her answer. "20%. It could be a bear. But it looks too short, and I think it's wearing a backpack."

"Very good," the teacher said. "I say 40%. It moves slowly for a bear, but it seems to me the right size."

"So I'm wrong," the student said. "It's 40%."

"No," the teacher replied. "You are perfectly right. For me, it is 40%, and for you, 20%."

"But you're the teacher. You know more."

"And your eyes are sharper than mine. Our perspectives are different, but neither is truer. I am right, and so are you."

"So is it a bear," the student said, with straining patience, "or not?"

The teacher closed her eyes. "What you seek is certainty. But a probability is only a perspective. Tell me, does that creature know whether it's a bear or not?"

"Of course."

"So for the creature itself, the probability must be 0% or 100%. It knows with certainty. You and I have our own perspectives, and thus our own probabilities." The teacher paused. "Tell me, if there were a full moon tonight, what would we see?"

"It'd be bright," the student said. "We could tell at a glance if that shadow is a bear."

"And if it were a new moon, what would we see?"

"Nothing. Darkness. There would be no shadow at all." The student paused. "We wouldn't see the creature approaching, so we wouldn't even be having this conversation."

"Precisely. When the moon is full and bright, we know all. There is no need for probability. And when the moon is new and dark, we know nothing, not even enough to ask a question. In either case – total knowledge, or total ignorance – probability is useless.

"Probability is for the nights like these," she continued. "It is for the nights of half-light. It is for the nights when we can make out a form, but cannot tell its precise shape. It is for nights when light and shadow mingle, when knowledge and ignorance share our thoughts. It is an expression of our uncertainty – no more, no less."

"So you're saying," the student said, "a probability depends on what we know, and what we don't know. And because you and I know different things, our probabilities are different."

The teacher smiled. Looking back out the window, the student found that the figure—bear, child, whatever it was—had vanished.

CH. 1: DISCUSSION

Probabilists list three approaches to probability: classical, empirical, and subjective.

"Classical" probability refers to chalkboard situations, like rolling dice and drawing cards, where we naively assume that a set of outcomes are equally likely. We take for granted, for example, that heads and tails are equally likely when flipping a coin—even though, in real life, a coin has microscopic imbalances, making one side slightly more likely than the other. Classical probability, then, is a purely theoretical game.

"Empirical" probability relies on real-world frequencies. For example, if 32% of skate-boarders broke their noses while attempting a trick, we give you a 32% probability of the same.

"Subjective" probability aims to express uncertainty in our minds – and it's much trickier to define. What does it mean to say that this shadow has a 20% chance of being a bear? There's only one shadow. Or that the President has a 74% chance of winning reelection? There's only one election. Or that tomorrow's chance of rain is 30%? There's only one tomorrow. Because we can't repeat these events, it's not obvious at first glance what such statements really mean.

CH. 1: QUESTIONS

1. Which approach to probability—classical, empirical, or subjective—rings truest to your intuitions about what probability is?

2. Most probability teachers (myself included, honestly) prefer to sweep "subjective probability" under the rug. Why do you think that is?

3. Why, then, would the teacher in our fable want to introduce the student to probability via the subjective approach?

4. After the student hears the teacher say "40%," should she adjust her own guess to something higher than 20%—like, say, 25% or 30%?

5. So the student says "20%," then the teacher says "40%," and then let's suppose the student says, "I change my answer to 25%." Should the *teacher* now change *her* answer? How long could this process go on? How long *should* it go on?

6. Suppose the student says the probability it's a bear is 20%, and the probability it's a child is 90%. What's the problem with this? How does this illustrate a danger of subjective probability?